

External Magnetic Fields in QFT: A Non-Perturbative Approach

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Abstract

A discussion of the influence of boundaries and scalar field interactions in the non-perturbative dynamics of fermions in an external magnetic field, along with their possible applications to condensed matter and cosmology, is briefly presented. The significance of the results for electroweak baryogenesis in the presence of an external magnetic field is indicated.

1 Introduction

In this talk I would like to present some recently found non-perturbative effects of external magnetic fields in quantum field theories with fermions. They are related to the so called Magnetic Catalysis (MC) [1], a universal phenomenon that can be understood as the generation, through the infrared dynamics of the fermion pairing in a magnetic field, of a fermion dynamical mass (fermion gap) at the weakest attractive interaction between fermions.

The discussion will be particularized to the following two problems: (1) the influence of boundaries in the dynamical generation of a fermion mass in the presence of a magnetic field and (2) the modification of the nonperturbative fermion dynamics due to fermion-scalar interactions, and its possible implications for baryogenesis in the presence of strong magnetic fields.

2 Boundary Effects

The influence of non-trivial boundaries on the MC was investigated by Ferrer, Gusynin and Incera [2], who considered the Nambu-Jona Lasinio (NJL) model in a locally flat space-time with topology represented by the domain $R^3 \times S^1$ and in the presence of an external constant magnetic field.

Starting from the NJL effective action

$$W(\sigma, \pi) = -\frac{N}{2G} \int d^4x (\sigma^2 + \pi^2) - i \text{Tr} \log [i\gamma^\mu D_\mu - (\sigma + i\gamma_5 \pi)], \quad (1)$$

where the composite scalar σ and π are defined by

$$\sigma = -\frac{G}{N} (\bar{\psi}\psi), \quad \pi = -\frac{G}{N} (\bar{\psi}i\gamma_5\psi), \quad (2)$$

one can investigate the generation of a fermion dynamical mass by determining the stationary points of W . Since the vacuum should respect translational invariance, it is enough to calculate the effective action for constant auxiliary fields. In this case, the effective action is just $W(\sigma, \pi) = -V(\sigma, \pi)T\mathcal{A}a$, where $T\mathcal{A}a$ is the space-time volume and V is the effective potential. In the proper-time formalism one gets

$$V_{AP}(\sigma) = \frac{N\sigma^2}{2G} + \frac{NeB}{8\pi^2} \int_{1/\Lambda^2}^{\infty} \frac{ds}{s^2} e^{-s\sigma^2} \theta_4 \left(0 \left| \frac{ia^2}{4\pi s} \right. \right) \coth(eBs), \quad (3)$$

$$V_P(\sigma) = \frac{N\sigma^2}{2G} + \frac{NeB}{8\pi^2} \int_{1/\Lambda^2}^{\infty} \frac{ds}{s^2} e^{-s\sigma^2} \theta_3 \left(0 \left| \frac{ia^2}{4\pi s} \right. \right) \coth(eBs), \quad (4)$$

for antiperiodic (*APBC*) and periodic (*PBC*) boundary conditions of the fermion fields at the boundary respectively[2].

The dynamical mass $\bar{\sigma}$ for the *PBC* case is the solution of the gap equation $dV_P/d\sigma = 0$. In leading order in $1/\Lambda$ the PBC gap equation is given by

$$\begin{aligned} \sigma & \left[\frac{1}{G} - \frac{\Lambda^2}{4\pi^2} + \frac{\sigma^2}{4\pi^2} \left(\log \frac{\Lambda^2}{\sigma^2} + 1 - \gamma \right) - \frac{1}{4\pi^2} \int_0^{\infty} \frac{ds}{s^2} e^{-s\sigma^2} \left(\theta_3 \left(0 \left| \frac{ia^2}{4\pi s} \right. \right) - 1 \right) \right. \\ & \left. - \frac{eB}{4\pi^2} \int_0^{\infty} \frac{ds}{s} e^{-s\sigma^2} \theta_3 \left(0 \left| \frac{ia^2}{4\pi s} \right. \right) \left(\coth(eBs) - \frac{1}{eBs} \right) + O\left(\frac{1}{\Lambda}\right) \right] = 0, \end{aligned} \quad (5)$$

where $\gamma \approx 0.577$ is the Euler constant. The corresponding gap equation for *APBC* is obtained by replacing θ_3 by θ_4 in Eq. (5).

It is easy to see that under the condition $1/a \ll \sigma \ll \sqrt{eB}$, i.e. a being the largest length scale in the problem, the gap equations for both cases (*PBC* and *APBC*) reduce to the following one

$$\sigma \left[\frac{1}{G} - \frac{1}{G_c} \pm \left(\frac{2\sigma}{\pi a^3} \right)^{1/2} e^{-\sigma a} - \frac{eB}{4\pi^2} \log \frac{eB}{\pi \sigma^2} \right] = 0, \quad (6)$$

where $G_c = (4\pi^2/\Lambda^2)$ and \pm refers to *APBC* and *PBC*, respectively. The solution of Eq. (6) in the $G \ll G_c$ approximation is

$$m_{dyn} \equiv \bar{\sigma} = \sqrt{\frac{eB}{\pi}} \exp \left(-\frac{2\pi^2}{eBG} \right). \quad (7)$$

As expected, the solution (7), which is nonanalytic in G as $G \rightarrow 0$, coincides with the one found in (3+1)-dimensions for $B \neq 0$ and $a = \infty$ [1].

Let us consider now the small length a ($\sigma, \sqrt{eB} \ll 1/a$) limit, which is the important one to study the effects of the compactified dimension. In this case the gap equation (5) for *PBC* reduces to

$$\sigma \left[\frac{1}{G} - \frac{1}{G_c} - \frac{1}{3a^2} + \frac{\sigma^2}{4\pi^2} \left(\log \frac{\Lambda^2 a^2}{16\pi^2} + \gamma \right) - \frac{eB}{2\pi\sigma a} \left(\sqrt{\frac{2}{eB}} \sigma \zeta\left(\frac{1}{2}, \frac{\sigma^2}{2eB} + 1\right) + 1 \right) \right] = 0, \quad (8)$$

where $\zeta(\nu, x)$ is the generalized Riemann zeta function.

As $B \rightarrow 0$, one recovers the known gap equation [3] which admits a nontrivial solution only if the coupling is supercritical, $G > G_c^a$, and the critical coupling $G_c^a = (G_c^{-1} + 1/3a^2)^{-1}$. When an external magnetic field, $B \neq 0$, is present a nontrivial solution exists at all $G > 0$ and, in particular, at $G \ll G_c^a$. Indeed, looking at the solution of Eq. (8) satisfying $\bar{\sigma} \ll \sqrt{eB}$, it is found

$$m_{dyn}^P \equiv \bar{\sigma} \simeq \frac{eB}{2\pi a} \frac{GG_c^a}{G_c^a - G} \quad (9)$$

if the coupling $G \ll G_c^a$. The condition $G < G_c^a$ guarantees that (9) is a minimum solution of the effective potential V_P .

Therefore a dynamical mass solution (9) exists in the weak coupling regime of the theory. The fact that there is no critical value of the coupling to produce chiral symmetry breaking is a characteristic feature of the catalysis of dynamical symmetry breaking by a magnetic field[1]. It is remarkable

that unlike the $a = \infty$ case, where the dynamical mass has nonanalytical dependence on the coupling constant as $G \rightarrow 0$, at finite a the dynamical mass (9) is an analytic function of G at $G = 0$.

Note also that $m_{\text{dyn}} = \langle 0 | \sigma | 0 \rangle = -G \langle 0 | \bar{\psi} \psi | 0 \rangle / N$. From here and Eq. (9) one finds that the condensate $\langle 0 | \bar{\psi} \psi | 0 \rangle$ is $\langle 0 | \bar{\psi} \psi | 0 \rangle = -N |eB| / 2\pi a$ in leading order in G ; i.e. it coincides with the value of the condensate calculated in the problem of free fermions in a magnetic field (see [2]). This point also explains why the dynamical mass m_{dyn} is an analytic function of G at $G = 0$: indeed, the condensate already exists at $G = 0$. As a result, there is a big enhancement of the dynamical fermion mass generation in the $R^3 \times S^1$ domain with periodic boundary conditions for the fermion fields as compared to the case of topologically trivial space-time (see Eqs. (9) and (7)).

Let us discuss now the *APBC* case. Following the same procedure used for *PBC*, it is easy to show that the *APBC* gap equation under conditions $\sigma, \sqrt{eB} \ll 1/a$ does not have a nontrivial solution; on the other hand, when $\sigma \ll \sqrt{eB}, 1/a$ it is reduced to

$$\begin{aligned} & \sigma \left[\frac{1}{G} - \frac{1}{G_c} + \frac{1}{6a^2} - \frac{eB}{4\pi^2} \int_0^\infty \frac{ds}{s} \theta_4 \left(0 \left| \frac{i}{4\pi s} \right. \right) \left(\coth(eBa^2s) - \frac{1}{eBa^2s} \right) \right. \\ & \left. + \frac{\sigma^2}{4\pi^2} \left(\log \frac{\Lambda^2 a^2}{\pi^2} + \gamma + eBa^2 \int_0^\infty ds \theta_4 \left(0 \left| \frac{i}{4\pi s} \right. \right) \left(\coth(eBa^2s) - \frac{1}{eBa^2s} \right) \right) \right] = 0. \end{aligned} \quad (10)$$

From Eq. (10) one can convince oneself that there is no nontrivial solution under the assumptions made if the coupling is weak ($G \rightarrow 0$). For chiral symmetry breaking to take place, the coupling constant G must be larger than some critical value that depends on the magnitude of the magnetic field B and size a . Indeed, Eq.(10) can be simplified in the limiting case $\sqrt{eB}a \gg 1$

$$\sigma \left[\frac{1}{G} - \frac{1}{G_c} + \frac{1}{6a^2} - \frac{eB}{4\pi^2} \left(\ln \frac{eBa^2}{\pi^3} + 2\gamma \right) + \frac{\sigma^2}{4\pi^2} \left(\log \frac{\Lambda^2 a^2}{\pi^2} + \frac{7\zeta(3)}{4\pi^2} eBa^2 + \gamma \right) \right] = 0. \quad (11)$$

From Eq. (11) one can notice that the magnetic field is helping the symmetry breaking since the critical coupling is less than the one corresponding to the case with zero magnetic field.

On the other hand, the contribution of the compactified dimension length a to the gap equation in the presence of a magnetic field has opposite sign

for *APBC* (third term in Eq. (10)), as compared to *PBC* (third term in Eq. (8)). Consequently, the boundary effect in the *APBC* case is not enhancing the chiral symmetry breaking, but on the contrary, it is counteracting it; while in the *PBC* case the magnetic catalysis is substantially enhanced by the boundary.

The behavior of the system in the *APBC* case can be better understood by realizing that due to the antiperiodic boundary conditions the quantity $1/a$ plays a role similar to temperature. Notice that chiral symmetry breaking takes place at small $1/a$ with a corresponding dynamical mass given by Eq. (7), so one should expect that at $1/a$ larger than some critical value $1/a_c$ the chiral symmetry must be restored.

Such a critical value $1/a_c$ indeed exists and is determined from the condition that the second derivative of the effective potential at $\sigma = 0$ becomes positive. In fact, from Eq. (11) it is found to be

$$\frac{1}{a_c} = \frac{e^\gamma}{\pi} m_{dyn} \quad (12)$$

with m_{dyn} the dynamical mass (7). Therefore, one obtains that the inverse of the critical length, $1/a_c$, is of the order of m_{dyn} , a result equivalent to that found for the relationship between the critical temperature and the gap in BCS superconductivity.

Finally, one should point out that the combined effect of an external magnetic field and non-trivial boundary conditions for fermions along third axis can find applications in condensed matter, in particular in high- T_c superconductors which are known to possess a quasi-2D structure (see discussion in [2]).

3 Scalar Interactions

Let us now investigate the influence of scalar fields in the non-perturbative dynamics of fermions in a magnetic field background [4]. With this goal in mind, let us consider a model field theory containing self interacting scalar fields, as well as Yukawa fermion-scalar interactions in the presence of an external constant magnetic field. As seen below, due to the magnetic field, a non-zero vacuum expectation value (vev) of the scalar field and a fermion dynamical mass arise as solution of the minimum equations of the system, breaking in this way the discrete chiral symmetry of the original Lagrangian.

However, in contrast to more conventional mechanisms to generate scalar vev's, the present model does not require the introduction of a scalar mass term with a wrong sign in the original Lagrangian, nor does it need dimensional transmutation "a la Coleman-Weinberg." Instead, the scalar vev is catalyzed, along with a fermion-antifermion condensate, by the external magnetic field. Both the fermion condensate and the scalar vev contribute to the dynamically generated fermion mass. The scalar vev grows with the square root of the magnetic field strength. No particular value of the magnetic field needs to be assumed, since there is no critical magnetic field for the zero-temperature symmetry breaking considered in the present work. The only assumption made is that all couplings are weak enough to justify a Hartree-Fock approach.

The starting point is a theory of gauge, fermion and scalar fields described by the following Lagrangian density

$$L = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\bar{\psi}\gamma^\mu\partial_\mu\psi + g\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{\lambda}{4!}\varphi^4 - \frac{\mu^2}{2}\varphi^2 - \lambda_y\varphi\bar{\psi}\psi \quad (13)$$

It has a U(1) gauge symmetry, a fermion number global symmetry, and the discrete chiral symmetry

$$\psi \rightarrow \gamma_5\psi, \quad \bar{\psi} \rightarrow -\bar{\psi}\gamma_5, \quad \varphi \rightarrow -\varphi \quad (14)$$

To study the vacuum solutions of the theory (13) under the influence of an external constant magnetic field B , we need to solve the extremum equations of the effective action Γ for composite operators[5]

$$\frac{\delta\Gamma(\varphi_c, \bar{G})}{\delta\bar{G}} = 0, \quad (15)$$

$$\frac{\delta\Gamma(\varphi_c, \bar{G})}{\delta\varphi_c} = 0 \quad (16)$$

where $\bar{G}(x, x) = \sigma(x) = \langle 0 | \bar{\psi}(x)\psi(x) | 0 \rangle$ is a composite fermion-antifermion field, and φ_c represents the vev of the scalar field. If the minimum solutions of (15) and (16) are different from zero the discrete chiral symmetry (14) will be broken and the fermions will acquire mass. We shall see that due to the magnetic field this is indeed the case.

The leading contributions to Eqs. (15) and (16) at large magnetic field, lead to the following minimum equations for the fermion mass m and the scalar vev respectively[4],

$$m \simeq m_0 + \left(\frac{g^2}{4\pi} - \frac{\lambda_y^2}{8\pi} \right) \frac{m}{4\pi} \ln^2 \left(\frac{gB}{m^2} \right) + \frac{1}{\pi^2} \frac{\lambda_y^2}{\lambda \varphi_c^2} gBm \ln \left(\frac{gB}{m^2} \right) \quad (17)$$

$$\frac{\lambda}{6} \varphi_c^3 + \frac{\lambda^2}{64\pi^2} \varphi_c^3 \left(\ln \left(\frac{\varphi_c^2}{gB} \right) - \frac{11}{3} \right) - \lambda_y \frac{gB}{2\pi^2} m \ln \left(\frac{gB}{m^2} \right) \simeq 0 \quad (18)$$

Assuming $\varphi_c \ll \sqrt{gB}$ (something that is corroborated by the results), the solution of these equation can be reduced to

$$m \simeq \frac{1}{\sqrt{\kappa}} \sqrt{gB} \quad (19)$$

$$\varphi_c \approx \frac{0.8}{\kappa^{1/2} \lambda_y} \sqrt{gB} \quad (20)$$

where the coefficient κ satisfies

$$\kappa \ln \kappa \simeq 1.4 \frac{\lambda}{\lambda_y^4} \quad (21)$$

Notice that the solutions (19) and (20) are indeed non-perturbative in the couplings constants. At each fixed scalar self-coupling λ , the values of m and φ_c increase with λ_y , since the parameter κ falls down much more rapid than $\frac{1}{\lambda_y}$ as λ_y increases.

It is a well known fact that in the absence of a magnetic field, the one-loop effective action (effective potential) of the present model would have a minimum at some non-trivial value of the scalar vev, but this minimum would lie far outside the expected range of validity of the one-loop approximation, even for arbitrarily small coupling constant, so it would have to be rejected as an artifact of the used approximation. In the present case however, thanks to the magnetic field, a consistent scalar field minimum solution is generated by non-perturbative radiative corrections. In this sense, a sort of non-perturbative Coleman-Weinberg mechanism takes place, with the difference that here no dimensional transmutation occurs. Since the theory already contains a dimensional parameter: the magnetic field B , there is no need to include scalar-gauge interactions in order to trade a dimensionless coupling for the dimensional parameter φ_c .

From the obtained results it can be seen that the scalar field interactions significantly enhance the magnetic catalysis. The enhancement of the dynamical mass due to the scalar field interactions is comparable to the effect produced by lowering the number of spatial dimensions in a theory that already exhibits magnetic catalysis in 3+1 dimensions [6].

A noteworthy feature of the results is that there is no trivial solution (stable or unstable) for the scalar vev in the present theory. Besides, no critical value of the magnetic field is required to produce the fermionic condensate and the scalar vev.

When studying this model at finite temperature, it is logical to expect that the increase in the dynamical mass due to the scalar field interactions will lead to a corresponding increase in the temperature at which the discrete symmetry is restored, since, typically, the critical temperature separating chiral broken-unbroken phases is of the order of the zero-temperature dynamical mass.

The substitution of real scalars by complex scalars does not lead to any qualitative change in the main results here discussed. However, the model with complex scalars better resembles a simplified version of the electromagnetic sector of the $SU(2) \times U(1)$ electroweak model (the scalar is not coupled to the electromagnetic field, just as the Higgs field of the electroweak theory). Since no tree-level scalar mass term with the wrong sign is introduced, the external magnetic field catalyzes gauge symmetry breaking by producing a non-trivial scalar vev through the non-perturbative mechanism shown in this paper. Even more, it is natural to expect that in the usual situation, where the gauge symmetry is broken in the standard way through the introduction of a wrong-sign scalar mass in the Lagrangian, the non-perturbative contributions induced by the external magnetic field will still yield an increment of the scalar field vev whose physical consequences are yet to be determined.

How significant for cosmology the non-perturbative effects here discussed are is still an open question. One can envision though that in the high temperature region of the electroweak theory, where the temperature-dependent fermion masses are nearly zero, these magnetic field-driven non-perturbative effects may be important at reasonable large magnetic fields. If that is the case, the scalar vev may be consequently visibly augmented (an effect up to now ignored by previous works). An interesting point would be then to determine whether the new magnetic field dependence of the scalar vev is large enough to change the recent conclusions on the role of magnetic fields in electroweak baryogenesis [7][8].

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